First, what are the units? In this example, the arrival amounts are measured in parcels, and time is measured in hours on a clock. That distinction matters: "12 parcels" is a total, while "12 parcels per hour" is a rate. They answer different questions.

Here is the clean version of the problem: *A warehouse receives 12 parcels at 9 am and 8 parcels at 10 am; how should those arrivals be represented over time?* The answer depends on what the numbers record. Are they batch sizes arriving at those exact times, or cumulative totals measured at those times? Those interpretations lead to different graphs and different calculations.

Start by Defining What Is Being Recorded

Before drawing a graph or calculating a rate, write down what the vertical axis represents. For example, it might show:

- **Parcels in each arrival batch:** 12 parcels arrive at 9 am, and 8 parcels arrive at 10 am.
- **Cumulative parcels received:** the total number received by 9 am is 12, and the total by 10 am is 8.
- **Arrival rate:** the number of parcels arriving per hour over specified time intervals.

The second interpretation has a potential issue: if arrivals continue, a cumulative total usually cannot fall from 12 at 9 am to 8 at 10 am. The first interpretation, however, is perfectly reasonable: a batch of 12 arrives at 9, followed by a batch of 8 at 10. The phrase "12 parcels at 9 am" should therefore be labeled clearly as a batch count unless the data source defines it another way.

In a workplace chart or dashboard, check the axis label, the report definition, and the time window used for each entry. "Parcels," "parcels per hour," and "cumulative parcels" are not interchangeable labels.

Model the Arrivals as a Step Graph

If 12 parcels arrive at 9 am and 8 more arrive at 10 am, a useful quantity to graph is the **cumulative number of parcels received**. Assume the warehouse has received none of these parcels before 9 am, and assume these are the only arrivals being modeled. The cumulative total is 0 before 9, becomes 12 at 9, and becomes 20 at 10.

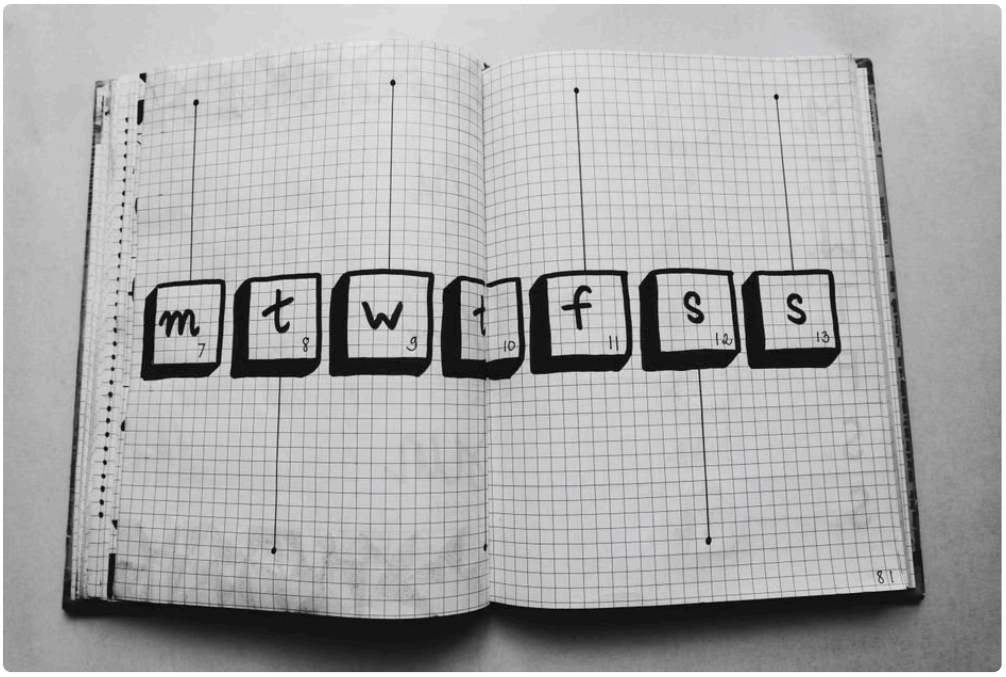
Time New parcels arriving Cumulative parcels received Before 9 am 0, under this example's assumptions 0 At 9 am 12 12 At 10 am 8 20

The cumulative graph is a **step graph**: it stays flat between recorded arrival times and jumps when a batch arrives. If we let t be the number of hours after 9 am, one possible model is

$$ C(t)= \begin{cases} 0, & t<0,\\ 12, & 0\le t<1,\\ 20, & t\ge 1, \end{cases} $$

where $C(t)$ is measured in parcels. This model treats the arrival at 9 am as included at $t=0$, and the arrival at 10 am as included at $t=1$. The exact rule at a jump matters when someone asks about the count *at* the event time. A chart should use a consistent convention and explain it when needed.

This is the right kind of **step graph warehouse** when the quantity is cumulative inventory received and arrivals happen in batches. The horizontal axis is time; the vertical axis is cumulative parcels. The flat parts mean "no additional recorded arrivals during this interval," under the example's assumptions. The jumps mean a batch was recorded.



Why Not Connect the Points with a Straight Line?

A straight line from the 9 am point to the 10 am point suggests that parcels arrived smoothly throughout the hour. But the information says arrivals happened at 9 and 10. It does not say that arrivals happened continuously in between. Drawing a sloping line can therefore tell a story the data never recorded.

Use a step graph when events occur in jumps or when the quantity stays constant between updates. A line graph may be appropriate for a continuously changing quantity, or when a report explicitly summarizes a value over time and interpolation is justified. For discrete shipments, a step graph usually preserves the timing more honestly.

There is an important distinction between the graph of *batch size* and the graph of *cumulative arrivals*. Batch size is 12 at 9 am and 8 at 10 am; it is not naturally a continuous line either. Cumulative arrivals, by contrast, provide the classic step graph: the running total increases at each arrival and holds steady between arrivals.

Totals and Rates Answer Different Questions

Suppose the question is, “How many parcels arrived in these two batches?” Add the batch sizes:

$$ 12\ \textparcels + 8\ \textparcels = 20\ \textparcels. $$



The answer is a **total**: 20 parcels. It is not a rate. A rate needs a time interval in its units, such as parcels per hour.

Now suppose the question is, "What was the average rate of increase in the cumulative total from 9 am to 10 am?" If the cumulative count is 12 at 9 and 20 at 10, then the change in count is $20-12=8$ parcels over 1 hour:

$$\textAverage rate of change =\frac20-12\ \textparcels10\text am-9\text am =\frac8\ \textparcels1\ \texthour =8\ \textparcels per hour.$$

This is an **average rate of change across an interval**. It does not mean that exactly 8 parcels arrived at every moment, or that arrivals were evenly spread through the hour. In this example, the 8 parcels are recorded as arriving at 10 am. The average rate summarizes the net change between the two endpoint measurements; it does not describe the detailed timing inside the interval.

Be especially careful with the phrase "12 parcels at 9 am, 8 parcels at 10 am." If those are batch sizes, the total is 20 parcels. But the average arrival rate from 9 to 10 cannot be determined from those two batch sizes alone without knowing whether the 9 am batch belongs to the interval, whether there were other arrivals, and exactly which interval the question means. If the 12-parcel batch arrived at 9 am and the 8-parcel batch arrived at 10 am, both are event counts at timestamps—not automatically a rate for the hour from 9 to 10.

Intervals Matter for Average Rate of Change

An average rate always compares a change in quantity with a change in time. In general, for a cumulative count $C(t)$, the average rate from time t_1 to time t_2 is

$$\fracC(t_2)-C(t_1)t_2-t_1.$$

The denominator is not a decorative detail. If $t_1=9$ am and $t_2=10$ am, the interval is one hour. If a dashboard instead compares 9:15 am with 10:00 am, the interval is 45 minutes, or 0.75 hour. Mixing the quantities without converting units can produce a misleading result.

For example, if a cumulative count increases by 8 parcels over 45 minutes, the average rate in parcels per hour is

$$\frac8\ \textparcels0.75\ \texthour \approx 10.67\ \textparcels per hour.$$

The total change is still 8 parcels. The rate is approximately 10.67 parcels per hour. Calling the total "10.67 parcels" would be wrong; calling the rate "8 parcels" would also be wrong. Units are a quick way to catch this kind of

mistake.

Define the Start Time and the Counting Rule

A model needs a clear time origin. You could define $t=0$ as 9 am, so $t=1$ is 10 am. Or you could use clock time directly. Either choice can work, but label it. Also decide what "by 9 am" means:

- Does it include a batch arriving exactly at 9:00?
- Is the count recorded just before the arrival or just after it?
- Does "9 am" mean an exact timestamp, or does it represent an hour-long reporting bucket such as 9:00–10:00?

These details can change how a count is assigned to a time interval. In a dashboard, "arrivals at 9 am" might mean events stamped at 9:00, or it might mean arrivals recorded during the 9 o'clock hour. Those are different data definitions. If the data is grouped into hourly buckets, label the bucket boundaries explicitly—for example, "9:00–9:59 am"—rather than implying a precise instant.

A Practical Workflow for Warehouse Data

1. **Ask what the units are.** Identify parcels, hours, minutes, or parcels per hour.
2. **Rewrite the statement clearly.** For example: "A batch of 12 parcels arrives at 9 am, and a batch of 8 arrives at 10 am."
3. **Name the recorded quantity.** Is the chart showing each batch size or the cumulative count?
4. **Set the start time.** State what $t=0$ means and whether event-time counts include the arrival at that instant.
5. **Choose the graph that matches the events.** Use a step graph for a cumulative total that jumps at arrivals; do not connect jumps with a slanted line unless the data supports continuous change.
6. **Calculate a rate only after choosing an interval.** Divide the change in parcels by the elapsed time, and keep the units in the answer.

Using MathJax and Runestone to Teach the Model

MathJax can render the piecewise definition of a step function and the average-rate formula clearly in a web article or activecalculus.org course page. This helps readers see both the intervals and the units in expressions such as $\frac8\ \textparcels1\ \texthour$. A formula is most useful when its variables and units are explained in nearby text.

Runestone is an interactive publishing platform that can support a short learning activity around the same example. For instance, learners could choose whether a graph should be a step graph or a straight line, identify the total number of parcels, and then calculate an average rate over a stated interval. Feedback can reinforce why "20 parcels" and "8 parcels per hour" are answers to different questions.

The Main Idea

For arrivals of 12 parcels at 9 am and 8 parcels at 10 am, first decide whether the figures are batch sizes or cumulative readings. If they are batch sizes, the two batches total 20 parcels. If you graph cumulative arrivals, the

count jumps at the recorded times, so a step graph is usually the honest choice. To calculate an average rate, specify the interval, compare the cumulative counts at its endpoints, and include time in the units.

Clear labels prevent the most common mistakes: treating a total as a rate, treating a timestamp as an interval, or drawing a line that implies arrivals happened between recorded events. Start with the quantity, the start time, and the units—and the right model becomes much easier to see.